International Journal of Scientific & Engineering Research, Volume 4, Issue 2, February-2013
ISSN 2229-5518

Strength and Weakness of labeled Graphs

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Abstract— In this paper a systematic mehod to find the number of strong and weak edges of a labeled graph is discovered.The method is applied on some labeled graphs.A family of weak graphs is discovered and foundout the Strength and weakness of 25 well known labeled Graphs.

Keywords— Graph labeling, Strong edge, Strong Graphs, Strength, and Weakness.

1 Introduction

Strong Graphs assure distinct numbers in ascending order to some edges and its end vertices. Labeled graph has application in the modelling and problems comes under the areas like Radar, Communication networks, Circuit design,Coding theory,Astronomy,X-ray,Crystallography,Database Management,Modelling of Constraint Programming over Finite domain,Bioilogy, chemistry ,etc.Same graph structure can have various labeling. According to the labeling the number of strong and weak edges varies. So a systematic method to find the number of strong and weak edges of a labeled graph has importance. In this paper the systematic method to find the strength and weakness of a labeled graph is discovered.The method is applied on some labeled graphs and the strength and weakness of 25 well known labeled graph is obtained.

2 Section I: Preliminaries

Definition 2.1: Graph labeling
A labeling of a Graph G=(V,E) is a one to one mapping ψ of the vertex set V(G) in to the non negative set of integers that induces for each edge e =uv ϵE(G), a label depending on the vertex labels ψ(u) and ψ(v).

Definition 2.2: Strong Graph
A labeled graph G=(V,E) is a strong graph if it satisfies the condition that there exist a number δ where $0 < \delta <$ Max {ψ(e)/e ϵ E(G) and ψ, the labeling} such that Min{ψ(u), ψ(v)}< ψ(uv) < Max {ψ(u), ψ(v)}
Note: 2.2.1: If the labeling is α then Max {ψ(e)/e ϵ E(G)}= | E(G)| and ψ(uv) = | ψ(v) – ψ(u) |

Definition 2.3: Strong edge
A strong edge e =uv of a strong graph is the edge which satisfies Min{ψ(u), ψ(v)}< ψ(u,v) < Max {ψ(u), ψ(v)}.

Definition 2.4: Weak edge
The edge of a labeled graph is weak if it is not strong.

Definition 2.5: Strength
Strength of a labeled graph is n{f^{-1}(1)} where f:ψ(u)xψ(v)→ {0,1} such that f(ψ(u),ψ(v)) =1 if e = uv is strong and 0 if e = uv is weak.

Note: 2.5.1: Strength of a labeled graph G is denoted by S(G).

Definition 2.6: Weakness
The weakness of a labeled graph is n{f^{-1}(0)} where f:ψ(u)xψ(v)→ {0,1} such that f(ψ(u),ψ(v)) =1 if e = uv is strong and 0 if e = uv is weak.

Note: 2.6.1: The weakness of a labeled graph G is denoted by W(G).

3 Section II: Strength and weakness

Theorem 3.1: S (G) +W (G) =| E (G) |

Proof: S(G) = n{f^{-1}(1)}
where f:ψ(u)xψ(v)→ {0,1} such that f(ψ(u),ψ(v)) =1 if e = uv is strong and 0 if e = uv is weak. This implies that S(G) = Number of strong edges of G.W(G) = n{f^{-1}(1)} where f:ψ(u)xψ(v)→ {0,1} such that f(ψ(u),ψ(v)) =1 if e = uv is strong and 0 if e = uv is weak. This implies that W(G) = Number of weak edges of G.

By definition 2.3 and 2.4, an edge is weak if its not strong. So if e ϵ E(G) then e is either strong or weak.
Therefore S(G)+W(G) = |E(G)|.

Theorem 3.2: In a labeled graph G, if u ϵ V (G) is such that ψ (u) =0 then W (G) $\geq$ deg (u).

Proof: Let deg (u) = k, k ϵ N. Then there exist k edges incident to u. Let u_i, i= 1,2,3,..,k be the vertices such that uu_i ϵ V(G) and n_i, i= 1,2,3,…,k be the labels such that $\psi(u_i) = n_i$ .Then Min{ψ(u), $\psi(u_i)$}= 0 and Max {ψ(u), $\psi(u_i)$} = n_i.This implies that all edges uu_i are weak.There fore W(G) $\geq$ k = deg(u).Hence the proof.

Lemma 3.2.1: In a simple connected labeled graph G, if ψ (u)= 0, for some u ϵ V(G) then S(G) < n.

Proof: If ψ (u)= 0,By theorem 3.2, W(G) $\geq$ k, k ϵ N.

To prove that k $\neq$0.
Suppose k=0, then deg(u)=0.This implies that u is an isolated vertex. But G being Simple connected labeled graph, it has no

isolated vertex. Hence $k \neq 0$.This implies that $\deg(u) \neq 0$.
i.e, $k > 0$.
But we know that $S(G) = n - W(G)$. That is $S(G) = n-k$, $k > 0$.This implies $S(G) < n$.

Lemma3.2.2: In a star G with centre u, and $\psi(u) = 0$, $S(G)=0$.
Proof: Suppose $\psi(u) = 0$.Being G a star $\deg(u)=n$. where $n=|E(G)|$.Therefore by theorem 3.1, $W(G) \geq n \rightarrow (1)$.
But $W(G) \leq n$ since $n=|E(G)|$.$\rightarrow(2)$. Equations (1) and (2) together implies that $W(G) = n$.
Now $S(G) = n-W(G)=n-n=0$.Hence the proof.

Theorem 3.3: $W(N^c) = |E(N^c)|$ where N^c is the complement of Nishad Graph.
Proof:First we shall prove that all the edges of N^c are weak edges.
Let n be the number of edges in a the complement of Nishad raph.As per definition of Nishad Graph ,it has a strong α labeling and (n-1) edges have common vertex u, such that $\psi(u) = n$ and the n^{th} edge vv^1 has α labeling $\psi(v) = n-2,\ \psi(v^1) = n-k$

Consider the complement of Nishad Ggraph

$$\psi^{\subset}(u) = n - n = 0$$
$$\psi^{\subset}(v) = n - (n-2k) = 2k$$
$$\psi^{\subset}(v^1) = n - (n-k) = k$$

Since n-1 edges have common vertex u, for all edges $uu^1 \in E(G), \psi^c(u) = 0$ and $\psi^c(u^1) = r_i$ where

$ri \in \{1,2,3...,n\} - \{k\}, i = 1,2,3...,n-1$

$\therefore$ For all $uu^1 \in E(u), \delta = \left|\psi^{\subset}(u) - \psi^{\subset}(u^1)\right| = ri$

$\therefore$ These edges $uu^1 \in E(u)$, are not strong edges. The graph will be strong only if vv'is strong but δ for vv' is

$$\delta = \left|\psi^{\subset}(v) - \psi^{\subset}(v^1)\right|$$
$$= |2k - k|$$
$$= k$$

Now $\psi^{\subset}(v^1) = \delta = k \therefore vv^1 is$ weak edge.

ie; The complement of Nishad graph doesnot contain a strong edge. This implies that all edges of the complement of Nishad Graph is weak.
Therefore the complement of Nishad graph is a weak graph.Hence $W(N^c) = |E(N^c)|$.
Lemma 3.3.1: If a graph G is weak then its weakness $W(G) = |E(G)|$ .The proof is obvious from definitions.

4 SECTION III: TABULAR REPRESENTATION

Let $u_0, u_0, u_1, \ldots, u_n$,be the vertices of a simple graph G=(V,E). Let $\psi(u_0), \psi(u_1), \psi(u_2), \ldots, \psi(u_n)$ be the vertex labels and $\psi(u_iu_j)$ be the edge labels for $i,j = 0,1,2,3,\ldots,n$. Assign 0 to $\psi(u_iu_j)$ if u_iu_j not an edge of G.Specify the labeling in the top left cell of the table . The following table gives the tabular representation of graph labeling of G.

G=(V,E) Labeling ψ		u_0	u_1	u_2	…..	u_n
		$\psi(u_0)$	$\psi(u_1)$	$\psi(u_2)$	…..	$\psi(u_n)$
u_0	$\psi(u_0)$	$\psi(u_0u_0)$	$\psi(u_0u_1)$	$\psi(u_0u_2)$	…..	$\psi(u_0u_n)$
u_1	$\psi(u_1)$	$\psi(u_1u_0)$	$\psi(u_1u_1)$	$\psi(u_1u_2)$	…..	$\psi(u_1u_n)$
u_2	$\psi(u_2)$	$\psi(u_2u_0)$	$\psi(u_2u_1)$	$\psi(u_2u_2)$	…..	$\psi(u_2u_n)$
…	…	…	…	…	…..	…
u_n	$\psi(u_n)$	$\psi(u_nu_0)$	$\psi(u_nu_1)$	$\psi(u_nu_2)$	…..	$\psi(u_nu_n)$

Example 4.1: The tabular representation of an α labeled N Graph N with $|E(G)|=5$ is given in the following table.

N=(V,E) α Labeling		u_0	u_1	u_2	u_3	u_4	u_5
		4	3	5	2	1	0
u_0	4	0	1	0	0	0	0
u_1	3	1	0	2	0	0	0
u_2	5	0	2	0	3	4	5
u_3	2	0	0	3	0	0	0
u_4	1	0	0	4	0	0	0
u_5	0	0	0	5	0	0	0

5 SECTION IV: PROPERTIES

Let $A=[\ \psi(u_iu_j)\ ]$, $i,j = 0,1,2,..,n$. be the matrix of tabular representation of G. Then A has the following properties.

1. All the main diagonal entries of A are zeros.
2. Every row of A has exactly 2 non zero entries if G is a cycle.
3. Every column of A has exactly 2 non zero entries if G is a cycle.
4. A is a symmetric matrix.i.e,$A=A^T$.

6 SECTION V: NISHAD'S METHOD

To find the strength and weakness of a labeled graph,I am suggesting the following method.

Nishad's Method:
Step 1: Represent the labeling in Tabular form.Let 2n be the number of non zero entries in the cells $\psi(u_iu_j)$.
Step 2: Delete the row and column corresponding to $\psi(u_i) = 0$.
Step 3: Delete the cells in the row of $\psi(u_i)$ if the entry in the cell $\psi(u_iu_j) \leq \psi(u_i)$.
Step 4: Count the number of cells $\psi(u_iu_j)$ from the remaining which satisfies $\psi(u_i) < \psi(u_iu_j) < \psi(u_j)$.Let it be k.

Then
Stength of G is $S(G) = k$ and Weakness of G is $W(G) = n-k$.

ISSN 2229-5518

7 SECTION VI: STRENGTH AND WEAKNESS USING NISHAD'S METHOD

Theorem 7.1: Let N be a Nishad Graph and |E(N)| =5, then S(N) = 2 and W(N) = 3.
Proof: We shall use Nishad's Method to prove this.
Step 1:
The tabular representation of the given N is

N=(V,E)		u_0	u_1	u_2	u_3	u_4	u_5
α Labeling		4	3	5	2	1	0
u_0	4	0	1	0	0	0	0
u_1	3	1	0	2	0	0	0
u_2	5	0	2	0	3	4	5
u_3	2	0	0	3	0	0	0
u_4	1	0	0	4	0	0	0
u_5	0	0	0	5	0	0	0

Note that there are 2n =10 cells $\psi(u_iu_j)$ have non zero entries.
Step 2:

N=(V,E)		u_0	u_1	u_2	u_3	u_4	u_5
α Labeling		4	3	5	2	1	0
u_0	4	0	1	0	0	0	
u_1	3	1	0	2	0	0	
u_2	5	0	2	0	3	4	
u_3	2	0	0	3	0	0	
u_4	1	0	0	4	0	0	
u_5	0						

Step 3:

N=(V,E)		u_0	u_1	u_2	u_3	u_4	u_5
α Labeling		4	3	5	2	1	0
u_0	4						
u_1	3						
u_2	5						
u_3	2			3			
u_4	1			4			
u_5	0						

Step 4:There are 2 cells which satisfies the condition
$\psi(u_i) < \psi(u_iu_j) < \psi(u_j)$.
Stength of N is S(N) = 2 and
Weakness of N is W(N) = 5-2=3.

Theorem 7.2: The strength and weakness of same graph structure need not be the same.
Proof: We shall prove it by using an example.

Case 1
Consider the graph C_8 .Let ψ be the α labeling of C_8 given by $\psi(u_0)=0$, $\psi(u_1)=5$, $\psi(u_2)=4$, $\psi(u_3)=6$, $\psi(u_4)=3$, $\psi(u_5)=7$, $\psi(u_6)=1$, $\psi(u_7)=8$.
We shall use Nishad's Method to find the strength and weakness of α labeling of C_8.

Step 1:The tabular representation of the given C_8 is

C_8-α		u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7
Labeling		0	5	4	6	3	7	1	8
u_0	0	0	5	0	0	0	0	0	8
u_1	5	5	0	1	0	0	0	0	0
u_2	4	0	1	0	2	0	0	0	0
u_3	6	0	0	2	0	3	0	0	0
u_4	3	0	0	0	3	0	4	0	0
u_5	7	0	0	0	0	4	0	6	0
u_6	1	0	0	0	0	0	6	0	7
u_7	8	8	0	0	0	0	0	7	0

Note that there are 2n =16 cells $\psi(u_iu_j)$ have non zero entries.
Step 2:

C_8-α		u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7
Labeling		0	5	4	6	3	7	1	8
u_0	0								
u_1	5		0	1	0	0	0	0	0
u_2	4		1	0	2	0	0	0	0
u_3	6		0	2	0	3	0	0	0
u_4	3		0	0	3	0	4	0	0
u_5	7		0	0	0	4	0	6	0
u_6	1		0	0	0	0	6	0	7
u_7	8		0	0	0	0	0	7	0

Step 3:

C_8-α		u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7
Labeling		0	5	4	6	3	7	1	8
u_0	0								
u_1	5								
u_2	4								
u_3	6								
u_4	3						4		
u_5	7								
u_6	1						6		7
u_7	8								

Step 4:There are 3 cells which satisfies the equation
$\psi(u_i) < \psi(u_iu_j) < \psi(u_j)$.
Stength of α labeling of C_8 is $S(C_8)$ = 3 and
Weakness of α labeling of C_8 is $W(C_8)$ = 8-3=5.

Case 2
Consider the graph C_8 .Let ψ be the complementary labeling of C_8 given by $\psi(u_0)=8$, $\psi(u_1)=3$, $\psi(u_2)=4$, $\psi(u_3)=2$, $\psi(u_4)=5$, $\psi(u_5)=1$, $\psi(u_6)=7$, $\psi(u_7)=0$.
We shall use Nishad's Method to find the strength and weakness of complementary labeling of C_8.

Step 1:The tabular representation of the given C_8 is

C_8- Complementary Labeling		u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7
		8	3	4	2	5	1	7	0
u_0	8	0	5	0	0	0	0	0	8
u_1	3	5	0	1	0	0	0	0	0
u_2	4	0	1	0	2	0	0	0	0
u_3	2	0	0	2	0	3	0	0	0
u_4	5	0	0	0	3	0	4	0	0
u_5	1	0	0	0	0	4	0	6	0
u_6	7	0	0	0	0	0	6	0	7
u_7	0	8	0	0	0	0	0	7	0

Note that there are 2n =16 cells $\psi(u_iu_j)$ have non zero entries.

ISSN 2229-5518

Step 2:

C_8-Complementary Labeling		u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7
		8	3	4	2	5	1	7	0
u_0	8	0	5	0	0	0	0	0	
u_1	3	5	0	1	0	0	0	0	
u_2	4	0	1	0	2	0	0	0	
u_3	2	0	0	2	0	3	0	0	
u_4	5	0	0	0	3	0	4	0	
u_5	1	0	0	0	0	4	0	6	
u_6	7	0	0	0	0	0	6	0	
u_7	0								

Step 3:

C_8-Complementary Labeling		u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7
		8	3	4	2	5	1	7	0
u_0	8								
u_1	3	5							
u_2	4								
u_3	2					3			
u_4	5								
u_5	1					4		6	
u_6	7								
u_7	0								

Step 4:There are 4 cells which satisfies the condition $\psi(u_i) < \psi(u_iu_j) < \psi(u_j)$.

Stength of Complementary labeling of C_8 is $S(C_8) = 4$ and Weakness of Complementary labeling of C_8 is$W(C_8) = 8-4=4$.

Case 3.

Consider the graph C_8 .Let ψ be the inverse labeling of C_8 given by $\psi(u_0)=4$, $\psi(u_1)=8$, $\psi(u_2)=0$, $\psi(u_3)=7$, $\psi(u_4)=1$, $\psi(u_5)=6$, $\psi(u_6)=3$, $\psi(u_7)=5$.

We shall use Nishad's Method to find the strength and weakness of inverse labeling of C_8 .

Step 1:

The tabular representation of the given C_8 is

C_8-Inversion Labeling		u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7
		4	8	0	7	1	6	3	5
u_0	4	0	4	0	0	0	0	0	1
u_1	8	4	0	8	0	0	0	0	0
u_2	0	0	8	0	7	0	0	0	0
u_3	7	0	0	7	0	6	0	0	0
u_4	1	0	0	0	6	0	5	0	0
u_5	6	0	0	0	0	5	0	3	0
u_6	3	0	0	0	0	0	3	0	2
u_7	5	1	0	0	0	0	0	2	0

Note that there are 2n =16 cells $\psi(u_iu_j)$ have non zero entries.

Step 2:

C_8-Inversion Labeling		u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7
		4	8	0	7	1	6	3	5
u_0	4	0	4		0	0	0	0	1
u_1	8	4	0		0	0	0	0	0
u_2	0								
u_3	7	0	0		0	6	0	0	0
u_4	1	0	0		6	0	5	0	0
u_5	6	0	0		0	5	0	3	0
u_6	3	0	0		0	0	3	0	2
u_7	5	1	0		0	0	0	2	0

Step 3:

C_8-Inversion Labeling		u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7
		4	8	0	7	1	6	3	5
u_0	4								
u_1	8								
u_2	0								
u_3	7								
u_4	1				6		5		
u_5	6								
u_6	3								
u_7	5								

Step 4:There are 2 cells which satisfies the condition $\psi(u_i) < \psi(u_iu_j) < \psi(u_j)$.Stength of Inverse labeling of C_8 is $S(C_8) = 2$ and Weakness of Inverse labeling of C_8 is $W(C_8) = 8-2=6$.

From case 1, case 2 and Case 3 it is obvious that the strength and weaknefulss of same graph structure need not be the same.

8 SECTION VII: FAMILY OF WEAK GRAPHS

During the investigation of strength and weakness using Nishad's Method,a family of some labeled graphs with strength 0 is discovered.It is obvious that there will exist more graphs having strength 0. In the theorems 8.1and 8.2,its proved that a family of graphs are weak.

Theorem 8.1: If n=2k, k ≥2 is even, then Edge Odd Graceful double star $K_{1,n,n}$ is a weak graph.

Proof: Let $V(K_{1,n,n}) = \{u, u_i, v_i; i=1,2,3,..,n\}$ and let $E(K_{1,n,n}) = AUB$ where $A=\{uu_i/i=1,2,3,\ldots,n\}$ and $B=\{u_iv_i/i=1,2,3,\ldots,n\}$.The EOGL is as follows.

$\psi(u)=0$
$\psi(uu_i)=2i-1$, i= 1,2,3,..,n
$\psi(u_iv_i)=2(n+i)-1$, i= 1,2,3,..,n
$\psi(u_i)=2(n+2i-1)(\text{mod } |4n|)$, i= 1,2,3,..,n
$\psi(v_i)= 2(n+i)-1$, i= 1,2,3,..,n

Now to prove that $K_{1,n,n}$ is weak.

It is enough to prove that the edge e=uv is weak for every $e \in E(K_{1,n,n}) = AUB$.Now we shall consider two cases.

Case 1.

Let $e \in A=\{uu_i/i=1,2,3,\ldots,n\}$
Then $e = uu_i$ for some $i \in \{1,2,3,\ldots,n\}$.
By the EOGL, $\psi(u)=0$
Therefore by Lemma 3.2.2, e is a weak edge.
Case 2.

Let $e \in B=\{u_iv_i/i=1,2,3,\ldots,n\}$
Then $e = u_i v_i$ for some $i \in \{1,2,3,\ldots,n\}$.
By the EOGL,

$\psi(u_iv_i)=2(n+i)-1$, for some $i \in \{1,2,3,\dots,n\}$.Let i=k.Then $\psi(u_kv_k)=2(n+k)-1$and $\psi(v_k)= 2(n+k)-1= \psi(u_kv_k)$.There fore the edge e doesnot satisfy the condition $\text{Minimum}(\psi(u_k), \psi(v_k)) < \psi(u_kv_k) < \text{Maximum}(\psi(u_k), \psi(v_k))$.This implies that e is a weak edge.Since this is true for every $k \in \{1,2,3,\dots,n\}$,e is weak for every $e \in B$.
Hence the proof.

Theorem 8.2: Edge odd graceful $K_{1,4,4}$,Complement of Nishad Graph,Graceful C_4, Even vertex graceful C_{11} and Edge odd graceful $B_{5,5}$ are weak Graphs.

Proof: To prove that Edge odd graceful $K_{1,4,4,}$ is weak.
It is enough to prove that $S(K_{1,4,4})=0$.We shall use Nishad's Method to prove this.

Step 1:
The tabular representation of the given $K_{1,4,4}$ is

$K_{1,4,4}$-EOGL		u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
		0	10	14	2	6	9	11	13	15
u_0	0	0	1	3	5	7	0	0	0	0
u_1	10	1	0	0	0	0	9	0	0	0
u_2	14	3	0	0	0	0	0	11	0	0
u_3	2	5	0	0	0	0	0	0	13	0
u_4	6	7	0	0	0	0	0	0	0	15
u_5	9	0	9	0	0	0	0	0	0	0
u_6	11	0	0	11	0	0	0	0	0	0
u_7	13	0	0	0	13	0	0	0	0	0
u_8	15	0	0	0	0	15	0	0	0	0

Note that there are 2n =16 cells $\psi(u_iu_j)$ have non zero entries.
Step 2:

$K_{1,4,4}$-EOGL		u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
		0	10	14	2	6	9	11	13	15
u_0	0									
u_1	10		0	0	0	0	9	0	0	0
u_2	14		0	0	0	0	0	11	0	0
u_3	2		0	0	0	0	0	0	13	0
u_4	6		0	0	0	0	0	0	0	15
u_5	9		9	0	0	0	0	0	0	0
u_6	11		0	11	0	0	0	0	0	0
u_7	13		0	0	13	0	0	0	0	0
u_8	15		0	0	0	15	0	0	0	0

Step 3:

$K_{1,4,4}$-EOGL		u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
		0	10	14	2	6	9	11	13	15
u_0	0									
u_1	10									
u_2	14									
u_3	2								13	
u_4	6									15
u_5	9									
u_6	11									
u_7	13									
u_8	15									

Step 4:There are 0 cells which satisfies the condition $\psi(u_i) < \psi(u_iu_j) < \psi(u_j)$.
Therefore
Stength of EOGL $K_{1,4,4}$ is $S(K_{1,4,4}) = 0$ and
Weakness of EOGL $K_{1,4,4}$ is $W(K_{1,4,4}) = 8-0=8$.
i.e, EOGL $K_{1,4,4}$ is a weak graph.

Similarly by using Nishad's Method, we shall prove that Graceful C_4, Even vertex graceful C_{11} and Edge odd graceful $B_{5,5}$ are weak Graphs.

Now to prove that Complement of Nishad Graph is a weak graph.

This statement is proved in Theorem 3.3.Hence the poof.

9 SECTION VIII: STRENGTH AND WEAKNESS OF 25 WELL KNOWN GRAPHS

Nishad's method is a good tool to analyse the strength and weakness of labeled graphs. Using Nishad's Method the strength and weakness of 25 well known labeled graphs is discovered.The result is displayed in the following table.

Strength and Weakness of some Labeled Graphs

Sl. No	Graph G	Labeling	S(G)	W(G)
1	N,|E(N)| =5	α	2	3
2	K_3	Graceful	1	2
3	C_4	Graceful	0	4
4	C_8	α	3	5
5	C_{15}	7 Graceful	12	3
6	W_7	3 Graceful	5	9
7	C_7	3 sequential	6	1
8	C_8	Complementary α	4	4
9	C_8	Inverse α	2	6
10	R_5	Graceful	4	6
11	R_6	α	5	7
12	H_5	Graceful	7	8
13	Δ_5 Snake	Graceful	9	6
14	$K_{1,6}$	3 sequential	6	0
15	French4 Windmill	Graceful	9	15
16	Dutch5 Windmill	Graceful	3	12
17	P_5xP_4	Even Graceful	18	13
18	P_{12}^1	Even Graceful	6	5
19	$P_{11}\Theta K_1$	Even graceful	13	8
20	$P_{12}\Theta K_1$	Even graceful	15	8
21	$K_{3,5}$	Even graceful	10	5
22	C_8UP_{12}	Odd Graceful	13	6
23	C_{11}	EvenVertex Graceful	0	11
24	$B_{5,5}$	EdgeOdd Graceful	0	11
25	$K_{1,4,4}$	EdgeOdd Graceful	0	8

International Journal of Scientific & Engineering Research Volume 4, Issue 2, February-2013
ISSN 2229-5518

10 Conclusions

Motivated by the definition of Graph labeling [4] , I have introduced Strong Graphs [1] and application of strong graphs in wireless networks [2] .In this paper the Strength and Weakness of labeled graphs is analysed. Introduced tabular representation of graph labeling and Nishad's method to find Strength and Weakness of labeled graphs.Proved that the strength and weakness of same graph structure need not be the same. A family of weak graphs is discovered .Also the Strength and Weakness of 25 well known labeled Graphs is discovered .The applications of Strong and weak Graphs in various fields of science and engineering is under investigation.

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